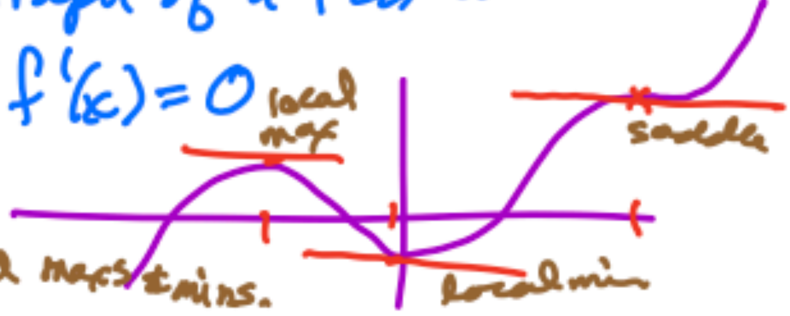
We have seen that

- The gradient vector $\nabla f(x,y) = (f_x, f_y, \ldots)$ ← This vector field is perpendicular to the contours (level sets) $f(x,y) = \text{constant}$.
- ∇f points in the direction where the function is increasing the fastest.
- $\|\nabla f\|$ = length of the gradient = rate of change of $f(x,y)$ as the point moves in that vector direction.

Calc 1: The tangent line of a graph of a fcn is horizontal when $f'(x) = 0$



x-values = critical points
→ helps us find maxs & mins.

Case 3: The tangent plane to the graph of a fcn is horizontal when $\nabla f(x,y) = (0,0)$.

Another use of ∇f:

Say a surface is described as the solution to the equation

$$g(x,y,z) = \text{const.} \quad \text{in } \mathbb{R}^3.$$

Then $\nabla g = (g_x, g_y, g_z)$ will be perpendicular to that, because it is a level set. Using this, we can find the equation of the tangent plane at a point.

Example

hyperboloid of one sheet.

Surface $x^2+y^2=z^2+4$

Question: What is the equation of the tangent plane to this surface at the point $(2,-1,1)$?

- Write surface as level set

$$g(x,y,z)=\text{constant}$$

$$g = x^2+y^2-z^2=4$$

plug in point

$$\nabla g = (2x, 2y, -2z) = (4,-2,-2)$$

= Normal to plane

point on plane $= (2,-1,1)$

$$N \cdot \left((x,y,z) - (2,-1,1) \right) = 0$$

$$\Rightarrow (4,-2,-2)(x-2,\ y+1,\ z-1) = 0$$

$$\Rightarrow \boxed{4(x-2) + -2(y+1) + -2(z-1) = 0}$$